**Translations of Important definitions or concepts of MCV4U, Rosedale Academy**

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**Calculus and Vectors / 微积分和向量**

**Unit 1: Rate of Change / 第一单元：变化率**

Instantaneous Rate of Change at a Point / 在某一点处的瞬时变化率

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| **Rate of Change:** How rapidly the dependent variable changes when there is a change in the independent variable.**(当自变量发生一定的变化时，因变量随之发生变化的快慢叫做变化率)** |

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| Now let’s consider a car moving 80 km in 1 hour. This value represents an **average rate of change**. This expression does not provide any information about the velocity of that car at different points during the hour. That rate you are travelling at a particular instant is called the i**nstantaneous rate of change**.**(**假如一辆车在一小时内行驶了80公里，那么这个数值代表的是**平均变化率。**我们不能根据这个80km/h而看出这辆车在这一小时内任意一点的速度。在某一瞬间的行驶速率叫做**瞬时变化率)** |

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| **Secants**: Lines that connect two points that lie on the same curve.(割线：连接在同一曲线上的两点的直线) |

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| **Tangents**: Lines that run parallel to, or in the same direction, as the curve, touching it at only one point, which is called **tangent point.**(**切线**：刚好触碰到曲线上某一点，并与曲线在这一点上的方向是相同或平行的直线。这个点叫做**切点**) |

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| For a simple curve like circle, a tangent is a line that intersects the circle once and only once. But this doesn't apply to complicated curves.(对于一些的曲线如同圆，一个切线是与圆仅交于一点的直线。但是这个并不在复杂的曲线中适用) |

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| A **linear function** is a function 'f' of the form f(x) = mx + b; where 'm' is the slope and 'b' is the y-intercept of the function.(一个**线性函数**是一个有着f(x) = mx + b 的形式的方程f；其中m是斜率，b是方程在y轴上的截距。) |

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| The slope of a line that passes through {P_1}({x_1},{y_1}) and {P_2}({x_2},{y_2}) is defined as: m = \frac{{\Delta y}}{{\Delta x}} = \frac{{{y_2} - {y_1}}}{{{x_2} - {x_1}}}一条穿过 {P_1}({x_1},{y_1})和 {P_2}({x_2},{y_2}) 的直线的斜率被定义为：m = \frac{{\Delta y}}{{\Delta x}} = \frac{{{y_2} - {y_1}}}{{{x_2} - {x_1}}} |

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| To determine the equation of a tangent to a curve at a given point, we first need to know the slope of the tangent. Consider a curve y = f(x) and a point P that lies on the curve. Now consider another point Q on the curve. The line joining P and Q is called a Secant. Now think of Q as a moving point that slides along the curve toward P, so that the slope of the secant PQ becomes a progressively better estimate of  the slope of the tangent at P.(要确定曲线在某一点的切线的方程式，我们首先需要知道切线的斜率。假设我们有一个曲线： y = f(x) ，和一个在曲线上的点 P 。在曲线上取另外一点 Q ， 穿过P 和 Q 的直线是一条割线。 现在将 Q 沿着曲线向 P点移动，割线 PQ 会越来越近似于曲线在 P点上的切线。) |

Real World Application of Rate of Change / 变化率在现实世界中的应用

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| Velocity as a Rate of Change / 速度作为变化率An object moving in a straight line is an example of a rate of change model. We use horizontal and vertical lines to represent the line of motion. For example movement to the right or up is considered as **positive direction** and movement to the left or down as **negative direction**. The average velocity of a car traveled 400 km in 4 hours would be 100 km/hr. Mathematically it is represented as:  V = \frac{{\Delta d}}{{\Delta t}}; where 'd' represents displacement and 't' represents time. (一个物体沿着一条直线的运动是一个变化率的模型。我们用水平方向上的直线或竖直方向上的直线来表示运动的方向。水平正右方向或竖直正上方向被设定为**正方向**，水平正左方向或竖直正下方向被设定为**负方向**。一辆在4小时内行驶了400公里的车的平均速度是100 km/hr。在数学上被表示为：  V = \frac{{\Delta d}}{{\Delta t}}；其中d 代表位移，t代表时间)  |

Introduction to Instantaneous Rate of Change / 瞬时变化率的介绍

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| **Instantaneous Rate of Change** is the measure of the rate of change for a continuous function at a point on the function. In other words, it refers to the rate of change at a specific point. It corresponds to the slope of the tangent passing through a single point, or tangent point, on the graph of a function.(**瞬时变化率**是一个连续方程在方程的某一点上的变化率。换句话说，它指代了某个特定的点上的变化率。它对应着穿过了这个在函数图像上的切点的切线方程的斜率。) |

Concept of Limits / 极限的概念

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| The notation is read "the limit of f(x) as 'x' approaches 'a' equals L" and means that the value of f(x) can be made arbitrarily close to L by choosing 'x' sufficiently close to 'a' (but not equal to 'a'). (符号 读作 "当x趋近于a时，f(x) 的极限等于L"。这代表着当x的取值无限趋近于a时（但不等于 'a'）， f(x) 的值将无限趋近于L).  |

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| **The limit** of a function y = f(x) at x = a is written as: Meaning that f(x) approaches the value of L as x approaches the value of a from both the left and right sides.   can be equal to f(a). In this case, the graph of f(x) passes through the point (a, f(a)). 函数y = f(x) 在 x = a 时的**极限**被记作： 意味着当x的取值从左侧或从右侧趋近于a时， f(x) 的值将趋近于L。当 f(x) 的函数图像穿过点 (a, f(a))时，  等于 f(a)。 |

**1. Sum Rule:** The limit of the sum of two functions is the sum of their limits.

 **加法法则：**两个函数之和的极限等于它们的极限之和。

**2. Difference Rule:** The limit of the difference of two functions is the difference of their limits.

 **减法法则：**连个函数之差的极限等于它们的极限之差。

**3. Product Rule:** The limit of a product of two functions is the product of their limits.

**乘法法则：**两个函数之积的极限等于它们的极限之积。

**4. Constant Multiple Rule:** The limit of a constant times a function is the constant times the limit of function.

**常数项乘积法则：**一个常数与函数的乘积的极限等于该常数与函数的极限之积。

**5. Quotient Rule:** The limit of the quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero.

**除法法则**：两个函数之商的极限等于它们各自的极限之商（当作为分母的极限不为0时）

**6. Power Rule:** The limit of the power of a function is the power of the limit of the function.

 **幂法则**：一个函数的n次幂的极限等于这个函数的极限的n次幂。

**7A. Root Rule:** The limit of a root of a function is root of the limit of the function.

**根式法则**：一个函数的n次方根的极限等于这个函数的极限的n次方根。（当根式存在时）

One-sided Limit / 单侧极限

The limit of a function exists at a point if both **the right-hand and left-hand limits** exist and they both approach the same value.

(如果函数在某一点的**左极限和有极限**存在且趋近于相同的数值，那么函数在这一点的极限存在)

A function f(x) is **continuous** at a number x = a if the following three conditions are

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| satisfied:**1.** f(a) is defined (that is 'a' is in the domain of the function)**2.** {\lim _{x \to a}}f(x) exists**3.** {\lim _{x \to a}}f(x) = f(a) If function 'f' is not continuous at 'a', we say 'f' is discontinuous at 'a'. |

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| 函数 f(x) 在 x = a 上连续的三个条件：**1.** f(a) 有意义(即 'a' 在函数的定义域内)**2.** {\lim _{x \to a}}f(x) 存在**3.** {\lim _{x \to a}}f(x) = f(a)如果函数 'f' 在 'a' 上不连续, 我们说 'f' 在 'a'上间断. |

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| * + All Polynomial functions are continuous for all real numbers.

 一切多项式函数在实数域上连续。* + A Rational function h(x) = \frac{{f(x)}}{{g(x)}} is continuous at x=a if g(a) \ne 0.

 当g(a) \ne 0时，有理函数h(x) = \frac{{f(x)}}{{g(x)}}在x=a上连续。* + A Rational function in simplified form has a discontinuity at the zeros of the denominator. A Point of Discontinuity is a point on the graph of a function that is not continuous.  It occurs when numerator and denominator of a rational function are simplified by dividing out the common factor.

一个有理函数在其最简形式的分母的零点上有间断。一个间断点存在于不连续的函数图像上。我们可通过将有理函数的分子和分母进行因式分解并将的公因式消掉，求出有理函数的最简形式，从而求出间断点的横坐标。* + When the one-sided limits are not equal to each other, then the limit at this point does not exist and the function is not continuous at that point.

当单侧极限不相等时（左右极限不相等），函数在该点处的极限不存在，且函数在该点上不连续。 |

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| **1. Instantaneous Velocity:**The velocity at a particular time t = a is calculated by finding the limiting value of the average velocity as h \to 0. The velocity of an object with position function s(t), at time time t = a, is: v(a) = {\lim _{\Delta t \to 0}}\frac{{\Delta s}}{{\Delta t}} = {\lim _{h \to 0}}\frac{{s(a + h) - s(a)}}{h} Note that the velocity v(a) is the slope of the tangent to the graph of s(t) at P(a, s(a))* + 1. **瞬时速度:**

通过求出h \to 0时平均速度的极限，我们可以算出在某一特定时间点 t = a 的速度。 某一物体的位置方程是 s(t), 其在时间t = a的速度是： v(a) = {\lim _{\Delta t \to 0}}\frac{{\Delta s}}{{\Delta t}} = {\lim _{h \to 0}}\frac{{s(a + h) - s(a)}}{h} 速度 v(a) 是函数 s(t) 的图像在点 P(a, s(a))上的切线的斜率。 |

Derivatives (导数):

This limit has two interpretations:

(1). the slope of the tangent to the graph  at point  and

(2). the instantaneous rate of change of  with respect to  at . Since this limit plays a central role in Calculus, we give it a special name as the derivative of  at  and notation as:

If you replace 'a' with the independent variable 'x', you arrive at the First Principles Definition of the derivative:

An alternate way of writing the definition of derivative is as follows:

(Translation) 极限的两种解释：

(1). 图像  在点  处切线的斜率，以及：

(2).  在 时的瞬时变化率。由于极限在微积分中起着重要作用， 我们给出导数的概念，即函数  在  处的导数，记作：

如果用自变量“x”来代替“a”，我们便得到导数的第一种定义：

另一种导数的定义是以下形式：

Leibniz Notation—莱布尼茨表达法（莱布尼茨表示法）

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| * The **derivative** of y=f(x) is a new function y=f'(x), which represents **the slope of the tangent** or instantaneous rate of change at any point on the curve of y=f(x).
* The derivative is defined by the first principles definition for the derivative or it can be represented by Leibniz notation.
* If the derivative does not exist at a point on the curve, the function is non-differentiable at that x-value. This can occur at points where the function is discontinuous or in cases where the function has an abrupt change.
* 函数 y=f(x) 的**导数**是一个新的函数 y=f'(x)。这个新的函数代表着原函数 y=f(x)上每一点的**切线的斜率**或每一点的瞬时变化率。
* 导数是以极限的方式定义的。可以用莱布尼茨表示法来标记导数。
* 如果在曲线上某一点上不存在导数，则称函数在那个点或在那个x的取值处“不可微”。这种情况出现在函数在某处不连续，或者函数在某处有着“非光滑的”或“突然的”变化（即函数图像在那一点是一个“尖锐的转折”。）
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Differentiable—可微的

Derivative of a Polynomial Function/多项式函数的导数：

1. The Constant Rule

2. The Power Rule

3. The Constant Multiple Rule

4. The Sum Rule

5. The Difference Rule

1. 常数法则：常数的导数为0.

2. 幂法则：x的n次方的导数为n倍的x的n-1次方：(xn)’=nxn-1

3. 常数项乘积法则：如果 , 则: 

4. 加法法则：两个函数的和的导数为这两个函数的导数的和。

5. 减法法则：两个函数的差的导数为这两个函数的导数的差。

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| The product rule（乘法法则）：(fg)' = fg' + f'gThe quotient rule（除法法则）：If h(x) = \frac{{f(x)}}{{g(x)}} then h'(x) = \frac{{f'(x).g(x) - f(x).g'(x)}}{{{{\left[ {g(x)} \right]}^2}}};g(x) \ne 0 |

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| The Chain Rule—链式法则：The chain rule tells us how to compute the derivative of a composite function. Just a recall given two functions 'f' and 'g', the composite function f \circ g is defined as f \circ g(x) = f\left( {g(x)} \right). If f and g are functions that have derivatives, then the composition function h(x) = f\left( {g(x)} \right) has a derivative given by:h'(x) = f'\left( {g(x)} \right).g'(x)链式法则告诉我们如何计算一个复合函数的导数。两个函数'f' 和 'g'的复合函数f \circ g 被定义为f \circ g(x) = f\left( {g(x)} \right)。 如果函数 f和g 都有导数，那么复合函数h(x) = f\left( {g(x)} \right) 的导数即为：h'(x) = f'\left( {g(x)} \right).g'(x) |

Trigonometric Function—三角函数；sine function—正弦函数；cosine function—余弦函数

Tangent function—正切函数；cotangent function—余切函数；secant function—正割函数

Cosecant function—余割函数

Exponential function—指数函数；logarithm function—对数函数