**Translations of Important definitions or concepts of MCV4U, Rosedale Academy**

*--By Owen Guo*

**Calculus and Vectors / 微积分和向量**

**Unit 2: Derivatives and their applications / 第二单元：导数及其应用**

2-1 Higher-Order Derivatives / 2-1 高阶导数

 **1. Displacement:** The distance and direction an object has moved from an origin over a period of time.

**2. Velocity:** The rate of change of displacement of an object with respect to time.

**3. Acceleration:** The rate of change of velocity with respect to time.

**1. 位移:** 物体从原点到在一段时间内移动到的位置的距离及方向。

**2. 速度:** 物体随着时间的位移变化率。

**3. 加速度:** 速度随着时间的变化率。

|  |
| --- |
| An object that moves along a straight line with its position determined by a function of time, s(t), has a velocity of v(t)=s'(t). In Leibniz notation, it is represented as:v = \frac{{ds}}{{dt}}The unit of velocity is displacement divided by time which is m/s.一个物体沿着一条直线运动，其位置可表达为时间的函数 s(t)，其速度则是v(t)=s'(t)。用莱布尼茨表示法记作：v = \frac{{ds}}{{dt}}速度的单位是位移除以时间，即为m/s。 |

Acceleration：加速度

Second derivative：二阶导数

|  |
| --- |
| The derivative of a derivative is called the Second Derivative. In the previous activity, we learned that if the position of an object, s(t), is a function of time t, then the first derivative of this function is called the velocity of the object with respect to time that is v(t) = s'(t) = \frac{{ds}}{{dt}}.Acceleration is represented by a(t). It is **the instantaneous rate of change** of velocity with respect to time. Acceleration is the first derivative of the velocity function and the second derivative of the position function.一个导数的导数被称为二阶导数。我们之前了解到，物体的位置是时间的函数s(t)，其一阶导数被称为物体的速度 v(t) = s'(t) = \frac{{ds}}{{dt}}。加速度记作a(t)，是速度随着时间的**瞬时变化率**。加速度是物体关于其速度的函数的一阶导数，是物体关于其位置的函数的二阶导数。 |

Higher Derivatives: 高阶导数

|  |
| --- |
| A second derivative of a function is determined by **differentiating** the first derivative of the function.一个函数的二阶导数是通过对其一阶导数**微分**而求出的。 |

* The derivative of a derivative function is called the Second derivative.
* If the position of the object  is a function of time , then the first derivative of this function represents the velocity of the object at time . 
* Acceleration, , is the instantaneous rate of change of velocity with respect to time. Acceleration is the first derivative of the velocity function and the second derivative of the position function. .
* 一个导函数的导数被称为二阶导数。
* 如果物体的位置可以表示为时间的函数，则其一阶导数是其速度的时间函数：
* 加速度, , 是速度随时间的瞬时变化率。加速度是速度方程的一阶导数，是位置方程的二阶导数：。

**Related Functions to Business:**

(1). The **demand function**, also known as price function, is p(x), where ‘x’ is the number of units of a product or service that can be sold at a particular price ‘p’.

(2). The **revenue function**, is , where 'x' is the number of units of a product or service sold at a price per unit of .

(3). The **cost function**, , is the total cost of producing 'x' units of a product or service.

(4). The **profit function**, , is the profit from the same of 'x' units of a product or service. The profit function is the difference between the revenue function and the cost function: .

**3. Derivatives of Business Function:**

Economists use the word 'marginal' to indicate the derivative of a business function.

(1).  or  is the **marginal revenue function** and refers to the instantaneous rate of change of total revenue with respect to the number of items sold.

(2).  or  is the **marginal cost function** and refers to instantaneous rate of change of total cost with respect to the number of items produced.

(3).  or  is the **marginal profit function** and refers to instantaneous rate of change of total profit with respect to the number of items sold.

***Translations:***

**和商学有关的函数：**

(1). **需求函数**, 也称价格函数, 表示为p(x), 其中 ‘x’ 是可以以特定价格‘p’销售的商品或服务单位的数量。

(2). **收益函数**, 表示为, 其中 'x' 是以为价格售出的商品或服务单位的个数。

(3). **成本函数**, , 是生产 'x' 个产品或服务单位的总花费。

(4). **利润函数**, , 是 'x' 个商品或服务单位对应的利润。利润函数是收益函数与成本函数之差： 。

**3. Derivatives of Business Function:**

经济学家用“边际”一词来表示一个商业函数的导数。

(1).  或  是**边际收益函数**，表示总收益随着售出商品数量的瞬时变化率。

(2).  或  是**边际成本函数**，表示总花费随着生产商品数量的瞬时变化率。

(3).  或   **边际利润函数**，表示总利润随着售出商品数量的瞬时变化率。

2-2 Extreme Values / 2-2 极限值（极值）

Increasing and Decreasing functions / 增函数和减函数

|  |
| --- |
| **Intervals:**There are certain sets of real numbers, called intervals, that occur frequently in calculus and correspond geometrically to line segments. For example, if a<b, the open interval from 'a' to 'b' consists of all numbers between 'a' and 'b' and denoted by the symbol (a,b).**区间:**在微积分中包含实数的集合被称作为区间，其几何意义是线的片断。例如，若a<b，则从 'a' 到 'b' 的开区间是由所有'a' 和 'b' 之间的数组成，并记作 (a,b)。 |

|  |
| --- |
| In general, a function f is called increasing on an interval I if: f({x_1}) < f({x_2})        whenever       {x_1} < {x_2} in interval I.  It is called decreasing on I if:f({x_1}) > f({x_2})        whenever       {x_1} > {x_2} in interval I.  函数 f 在区间 I 上被称作递增，如果:  在区间 I 中，  {x_1} < {x_2} 对应 f({x_1}) < f({x_2})。 在 I 上递减，如果:在区间 I 中， {x_1} > {x_2} 对应 f({x_1}) > f({x_2})。 |

|  |
| --- |
| **Test for Increasing and Decreasing Functions：****1.** If f'({x_1}) > 0 for all x in an interval I, then f is increasing on I.   **2.** If f'({x_1}) < 0 for all x in an interval I, then f is decreasing on I. **如何判断增减函数:****1.** 若对于所有在区间 I上的 x, 我们有f'({x_1}) > 0 ，那么 f 在 I上单调递增。   **2.** 若对于所有在区间 I上的 x，我们有 f'({x_1}) < 0 ，那么 f 在 I上单调递减。 |

Maximum and minimum values / 极大值和极小值：

Given a graph of a function , a point is a **local maximum** if the y-coordinates of all the points in the vicinity are less than the y-coordinate of the point. Algebraically, if  changes from positive to zero to negative as  increases from  to , then  is a **local maximum** and  is a **local maximum value**.

Similarly, a point is a **local minimum** if the y-coordinates of all the points in the vicinity are greater than the y-coordinate of the point. Algebraically, if  changes from negative to zero to positive as increases from  to , then  is a **local minimum** and  is a **local minimum value**.

Local maximum and minimum values of a function are called local extreme values,**local extrema**, or turning points. A function has an absolute maximum at  if  for all in the [domain](http://moodle.rosedaleacademy.com/mod/glossary/showentry.php?eid=1040&displayformat=dictionary). The maximum value of the function is . The function has an **absolute minimum** at  if for all  in the [domain](http://moodle.rosedaleacademy.com/mod/glossary/showentry.php?eid=1040&displayformat=dictionary). The minimum value of the function is .

***Translations:***

给出一个函数 的图像，如果一个点周边的所有点的纵坐标值都小于该点的纵坐标值，那么这个点被称作**局部极大**。从代数学的角度来看，如果  随着  由  增加到  而由正值变到零再变到负值，那么 是一个**局部极大**， 是一个**局部极大值**。

同样地，如果一个点周边的所有点的纵坐标值都大于该点的纵坐标值，那么这个点被称作**局部极小**。从代数学的角度来看，如果  随着  由  增加到  而由负值变到零再变到正值，那么 是一个**局部极小**， 是一个**局部极小值**。

一个函数的局部极大值和局部极小值被称作**局部极值**，这些点被称作转折点。若对于所有定义域上的x，函数满足，则称函数在x=a处有着一个绝对极大值，这个极大值则为 。 若对于所有定义域上的x，函数满足，则称函数在x=a处有着一个**绝对极小值**，这个极小值则为 。

"If function  has a local maximum or minimum at , then either  or  does not exist."

In general, **Fermat's Theorem** says that we should at least start looking for extreme values (maximum or minimum) at the numbers  for which . These numbers are called **Critical Numbers**.

A critical number of a function is a value  in the [domain](http://moodle.rosedaleacademy.com/mod/glossary/showentry.php?eid=1040&displayformat=dictionary) of the function for which either  or  does not exist. If  is a critical number, the point  is a critical point. To determine the absolute maximum and minimum values of a function in an interval, find the critical numbers, then substitute the critical numbers and also the x-coordinates of the endpoints of the interval into the function.

***Translations:***

"如果函数  在x=处有局部极大值或局部极小值，则 或  不存在”

**费马定理**告诉我们要至少从满足的点开始找极值。极值对应的横坐标上的数值(x=c)被称作**临界值**。

一个函数的临界值是在其定义域上的某一值 ，满足 或者  不存在。 若 是一个临界值，点  则是一个临界点。为了求出函数在某一区间的绝对极大值和绝对极小值，我们先找出临界值，并将它们代入函数中进行计算。我们也将该区间的两个末端端点代入到函数中进行计算。

**How to Determine Absolute Maximum and Minimum Values of a Function:**

We can apply the following procedure to find the absolute maximum and minimum values of a function in an interval :

**Step 1:** Take the derivative of the function .

**Step 2:** Find the critical numbers by finding the values of  that is .

**Step 3:**Find the values of  at the critical numbers of  in the interval .

**Step 4:** Find the values of  at the endpoints; that is evaluate  and .

**Step 5:** The largest of the values from step 3 and 4 is the absolute maximum value; the smallest of these values is the absolute minimum value.

**如何求一个函数的绝对极大值和绝对极小值:**

我们可以通过以下步骤来寻找一个函数在区间上的绝对极大值和绝对极小值:

**步骤 1:** 求 的导数。

**步骤 2:** 令 ，并求出满足该等式的x的值，这些值便是临界值。

**步骤 3:**将在区间的临界值带入函数中来求出对应的函数值。

**步骤 4:** 求出区间末端端点所对应的函数值，即求出  和 。

**步骤 5:** 从步骤3到步骤4中，我们所求出的最大的函数值即为绝对极大值，最小的函数值即为绝对极小值。

Optimization / 最优化

Concavity / 凹凸性

Asymptote / 渐近线：Horizontal Asymptote / 水平渐近线；Vertical Asymptote / 竖直渐近线；Oblique Asymptote / 斜渐近线。

Points of Inflection / 拐点

**Test for Concavity:**

* If  for all values of  in interval , then the graph is concave upward on .
* If  for all values of  in interval , then the graph is concave downward on .

Therefore it follows from this test that there will be a point of inflection at any point where the second derivative changes sign.

**凹凸性的判断:**

* 若区间上所有x值满足  ，则函数图像在上是上凹的。
* 若区间上所有x值满足  ，则函数图像在上是下凹的。

所以通过这样的判断，我们可以找到二阶导数发生符号变化的点（或二阶导数为零的点），即为拐点。

|  |
| --- |
| **Second Derivative Test:*** If **f'(c)=0** and **f''(c)>0**, then **f** has a local MINIMUM at **c**.
* If **f'(c)=0** and **f''(c)<0**, then **f** has a local MAXIMUM at **c**.
* If **f''(x)=0** and **f''(x)** changes sign at **c**, then there is a point of inflection at **(c, f(c))**.

**二阶导数判别法:*** 若 **f'(c)=0** 且 **f''(c)>0**, 则 **f** 在 **c** 处有一个局部极小值。
* 若 **f'(c)=0** 且 **f''(c)<0**, 则 **f** 在 **c** 处有一个局部极大值。

若 **f''(x)=0** 且 **f''(x)** 在 **c** 处变号，则 **(c, f(c))** 为拐点。 |

**Algorithm for curve sketching:**

**a) Domain:** Find the [domain](http://moodle.rosedaleacademy.com/mod/glossary/showentry.php?eid=1040&displayformat=dictionary) of the function. This will be useful when finding vertical asymptotes and determining critical numbers.

**b) Intercepts:** Find the x and y intercepts of the function, if possible. To find the x-intercept, we set y = 0 and solve the equation for x. Similarly, we set x = 0 to find the y-intercept.

**c) Symmetry:**Determine whether the function is an [odd function](http://moodle.rosedaleacademy.com/mod/glossary/showentry.php?eid=1039&displayformat=dictionary), an [even function](http://moodle.rosedaleacademy.com/mod/glossary/showentry.php?eid=1038&displayformat=dictionary) or neither.

**d) Asymptotes:**Find the asymptotes of the function using the methods described above. First attempt to find the [vertical asymptote](http://moodle.rosedaleacademy.com/mod/glossary/showentry.php?eid=1035&displayformat=dictionary) and [horizontal asymptote](http://moodle.rosedaleacademy.com/mod/glossary/showentry.php?eid=1036&displayformat=dictionary) of the function. If necessary, find the slant asymptote.

**e) Intervals of Increase and Decrease:**Determine the first derivative of the function **f'(x)** and the intervals where the function is increasing and decreasing.

**f) Local Maximum/Minimum:** Find the critical numbers of the function. Remember that the number c in the [domain](http://moodle.rosedaleacademy.com/mod/glossary/showentry.php?eid=1040&displayformat=dictionary) is a critical number if **f'(c) = 0** or **f'(c)** does not exist. Use the first derivative test to find the local maximums and minimums of the function. It is also possible to use the second derivative test.

**g) Concavity and Points of Inflection:** Determine the second derivative **f''(x)** and identify the intervals where the function is concave upward and concave downward. Inflection points occur whenever the curve changes in concavity.

**h) Sketch:**Using the information obtained in all the above steps, we can now sketch the curve. First, we draw dashed lines for the asymptotes of the function. Then plot the x- and y-intercepts, maximum and minimum points and points of inflection on the graph. Sketch the curve between the points, using the intervals of increase and decrease and intervals of concavity. Be sure that the graph behaves correctly when approaching asymptotes.

**曲线构图的步骤:**

**a) 定义域:** 找到函数的定义域。这有利于寻找竖直渐近线和临界值。

**b) 截距:** 如果存在，找到函数的横轴截距和纵轴截距。我们让 y = 0 来计算出横轴截距，让x = 0 来计算纵轴截距。

**c) 对称性:**判断函数是否为奇函数还是偶函数。

**d) 渐近线:**用以上所描述的方法来寻找函数的渐近线。先寻找竖直和水平渐近线，如有需要，再寻找斜渐近线。

**e) 增区间和减区间:**通过函数的一阶导数**f'(x)** 来判断其增区间和减区间。

**f) 局部极大值和局部极小值:** 先找出函数的临界值。函数定义域中的c值是临界值，如果其满足**f'(c) = 0** 或 **f'(c)** 不存在。用一阶导数判别法来寻找函数的绝对极大值和绝对极小值。如有需要，用二阶导数判别法。

**g) 凹凸性和拐点:** 求出函数的二阶导数 **f''(x)**并判断函数上凹和下凹的区间。当凹凸性在某点发生变化时，该点为拐点。

**h) 作图:**利用以上的信息，我们可以构图了。首先我们用虚线将函数的渐近线绘出。然后描出横截距、纵截距、极大值点、极小值点和拐点。根据增减区间和关于凹凸性的区间在以上所描绘出的点之间连接曲线，并确保函数在临近渐近线时的正确图像。

Polynomial Functions / 多项式函数

Rational Functions / 有理函数