**Translations of Important definitions or concepts of MCV4U, Rosedale Academy**

*--By Owen Guo*

**Calculus and Vectors / 微积分和向量**

**Unit 3: Geometry and Algebra of Vectors / 第三单元：向量的几何与代数**

3-1: Geometric Vectors / 几何向量

|  |
| --- |
| A **vector** is a mathematical object with both **magnitude (length)** and **direction**. It can be represented geometrically by a directed line segment, that is, a line segment with an arrow on one end to indicate the intended direction. The end of the line segment with no arrow is called the **tail** or **initial point** of the vector. The end with the arrow is its **tip** or **head** or **terminal point**.一个**向量**是一个拥有**大小（或长度）**和**方向**的数学对象。其几何意义是一个拥有方向的线段，用一个一端带有箭头的线段来表示，箭头的方向即代表向量的方向。线段没有箭头的一端叫做向量的**尾部**或者向量的**初始点**。带有箭头的一端叫做**尖端**或**首部**或**终止点**。 |

**Opposite vectors / 反向量（相反向量）**

**Addition and Subtraction of Vectors / 向量的加法与减法：**

**The Triangle Law for Adding two Vectors:**

|  |
| --- |
| For vectors  {\overrightarrow u }  and   {\overrightarrow v }, the sum (or resultant) of  {\overrightarrow u }  and   {\overrightarrow v }   is a vector fromthe tail of   {\overrightarrow u }   to the tip of   {\overrightarrow v },   when the tail of   {\overrightarrow v }  is placed at the tip of   {\overrightarrow u }.The sum is represented by  \overrightarrow u +\overrightarrow v .This method is also known as **tip-to-tail method**.***Translations:*** |

**向量相加的三角法则：**

|  |
| --- |
| 对于向量 {\overrightarrow u }  和 {\overrightarrow v }, 当向量 {\overrightarrow v }的尾部被放置在{\overrightarrow u }的尖端处，{\overrightarrow u }  与   {\overrightarrow v }  之和（或合量）是连接向量 {\overrightarrow u }  的尾部至向量   {\overrightarrow v } 的尖端的向量，记作 \overrightarrow u +\overrightarrow v .此方法叫做“**首尾相接法**”。 |

**The Parallelogram Law for Adding two Vectors:**

|  |
| --- |
| To determine the sum (or resultant) of the two vectors   \overrightarrow u    and   \overrightarrow v ,complete the parallelogram formed by these two vectors when placed tail to tail.Their sum is the vector   \overrightarrow {AD} , which is the diagonal of the constructed parallelogram. Consider the triangle formed by vectors    \overrightarrow u ,    \overrightarrow v ,   and   \overrightarrow u+\overrightarrow v .It is important to note that the magnitude of the sum or resultant is less than or equal to the combined magnitudes of both vectors. The magnitude of the sum is equal to the sumof the magnitudes of the vectors only when these three vectors lie in the same direction.  |

***Translations:***

**两向量相加的平行四边形法则：**

|  |
| --- |
| 欲算出两向量 \overrightarrow u    和  \overrightarrow v 的和（或合量），将向量的尾部放置在一起，并以该两向量为平行四边形的两边作出整个平行四边形。其合向量便是平行四边形中连接两向量重合的尾部的那条对角线：\overrightarrow {AD} 。 在由向量\overrightarrow u ,    \overrightarrow v ,   及   \overrightarrow u+\overrightarrow v 形成的三角形中，向量\overrightarrow u ,    \overrightarrow v 之和的模长等于或小于向量\overrightarrow u 的模长与向量  \overrightarrow v 的模长之和。向量\overrightarrow u ,    \overrightarrow v 之和的模长等于向量\overrightarrow u 的模长与向量  \overrightarrow v 的模长之和，当且仅当这三个向量的方向都相同。 |

**The Difference of two Vectors:**

The difference between vectors      and      is found by adding the opposite vector    to      using the triangle law of addition. Another way to think about      is to arrange the vectors tail to tail. In this case,  is the vector that must be added to     to get   .

***Translations:***

**两向量之差：**

向量     与    之差可通过向量   的反向量与向量    的三角相加法则求出。另一种考虑方式是将两向量的尾部相接，然后  则成为需要加至向量 从而求得  的向量（向量之和的逆向思维）。

**Null vectors or zero vectors / 零向量**

**Equilibrant Vector:**

|  |
| --- |
| An equilibrant vector is one that balances another vector or a combination of vectors.It is equal in magnitude but opposite in direction to the resultant vector. If the equilibrant isadded to a given system of vectors, the sum of all vectors, including the equilibrant, is NULL vector.In other words, the equilibrant is the opposite force, that is, the force that would exactlycounterbalance the resultant force. |

***Translations:***

**平衡向量:**

|  |
| --- |
| 一个平衡向量是与另外一个向量或者合向量平衡的向量，即长度或大小与合向量相同、方向与合向量相反的向量。如果一个已知系统的所有向量之和对应的平衡向量被加入该系统，则新的合向量是零向量。在力学中，平衡力是使系统平衡的与已知合力大小相同、方向相反的力。 |

## Multiplication of a Vector by a Scalar / 向量的标量乘积

|  |
| --- |
| When you multiply a vector by a scalar, the magnitude is multiplied by the scalar and the vector is parallel to the original vector. The direction remains unchanged if the scalar is positive, and becomes opposite if the scalar is negative.当用一个标量k乘以向量的时候，向量的模长变成了原来的k倍，并且该向量与原向量共线。若标量为正，则向量方向不变。若标量为负，则向量方向变为原来的反方向。 |

|  |
| --- |
| Multiplying  \overrightarrow a   by a different value of k can affect the direction and magnitude of a vector, depending on the values of k that are chosen.Given a vector   \overrightarrow v   and a scalar k, where k is a real number, the scalar multiple of k and   \overrightarrow v ,  is a vector  k\overrightarrow a   and its magnitude is  k\left| {\overrightarrow a } \right|.  * If  k > 0, then  k\overrightarrow a   has the same direction to  \overrightarrow v .
* If  k < 0, then  k\overrightarrow a   has the opposite direction to  \overrightarrow v .

向量  \overrightarrow a   与标量 k 之积的方向与大小与 k 值有关。给定一个向量   \overrightarrow v   和一个标量k（k 为某一实数）, k与 \overrightarrow v 的标量乘积, 是向量 k\overrightarrow a  ，其大小为 k\left| {\overrightarrow a } \right|.  * 若  k > 0, 则  k\overrightarrow a   与  \overrightarrow v  方向相同.

若 k < 0, 则  k\overrightarrow a   与  \overrightarrow v  方向相反. |

|  |
| --- |
| **Collinear Vectors:**When two vectors are parallel or lie on the same straight line, these vectors are described as being **collinear.** They are described as collinear because they can be translated so that they lie in the same straight line. Two vectors {\overrightarrow u }  and   {\overrightarrow v }  are collinear if and only if it is possible to find a **nonzero**scalar k such that \overrightarrow u=k\overrightarrow v .When discussing vectors, the terms parallel and collinear are used interchangeably.**共线向量:**当两个向量平行或在同一条直线时，我们称这两个向量**共线**。对于共线向量，我们能将它们进行移动、放置在同一条直线上。 两个向量 {\overrightarrow u } 和 {\overrightarrow v }  共线当且仅当我们能找到一个**非零**实数 k （在这里充当标量）以至于\overrightarrow u=k\overrightarrow v 。当讨论向量时，“平行”和“共线”两个词等价。 |

**Key Properties:**

* **Distributive Property:**For any scalar  (where  belongs to real numbers) and any vectors    and : k( + ) = k + k .
* **Associative Property:**For any scalar  and  (where  and  are real numbers) and any vector  : ab( ) = a(b ).
* **Identity Property:** For any vector  : 1( ) =  

***Translations:***

**关键性质:**

* **分配律:**对于任意实数  以及任意两个向量 和 ，我们有: k( + ) = k + k 。
* **结合律:**对于任意两个实数  和  以及任意一个向量 ，我们有: ab( ) = a(b )。
* **恒等性质:** 对于任意向量 ，我们有: 1( ) =  

**Resolutions of Vectors / 向量的分解**

**Linear combination of Vectors / 向量的线性结合**

|  |
| --- |
| **Resolution of a vector into Rectangular Components / 向量的矩形分量的分解**A vector can be resolved into two perpendicular vectors whose sum is the given vector. For example, both vertical and horizontal forces are acting on an object. These are called the Rectangular Components of the force.一个向量可以被分解成两个互相垂直的分向量，这两个分向量之和即为该向量。例如一个竖直方向的力和一个水平方向的力被施加在一个物体上。这些被叫做力的矩形分量。 |

**Cartesian Vectors / 笛卡尔向量**

One of the most important ideas that we must consider is that the unique representation of vectors in the xy-plane. This unique representation of the vector    is a matter of showing the unique representation of the point because    is determined by this point. The uniqueness of vector representation will be first considered for the position vector  , which has its head at the point    and its tail at the origin   .

The point  is a distance of  units away from the origin on x-axis and  units away from the origin along y-axis.

We define this special Cartesian vector as the position vector  ![\left[ {a,b} \right]](). We use square brackets to distinguish between the point    and the related position vector   ![\left[ {a,b} \right]]().

  is also used to represent any vector with the same magnitude and direction as .

***Translations:***

向量在xy坐标平面的代数表示也是本单元极其重要的概念。通过  这种独特的向量表示法，我们也能知道点  独特的位置，因为  由该点的位置决定。位置向量  是一个向量的独特表示法，其首部在点 、尾部在原点 。

点  在横坐标轴方向上距离原点有 个单位，在纵坐标轴方向上距离原点有个单位。

我们将笛卡尔向量定义为位置向量 ![\left[ {a,b} \right]]()。我们用方括号来代表位置向量，以区别于点坐标  。

 也用来表示与 有着相同模长和方向的其他向量。

**The Unit Vector:**

|  |  |
| --- | --- |
| The unit vectors  \overrightarrow i   and  \overrightarrow j    are the building blocks for Cartesian vectors. Unit vectors have magnitude 1 unit.  \overrightarrow i   and   \overrightarrow j    are special unit vectors that have their tails at the origin.The head of vector  \overrightarrow i   is on the x-axis at (1, 0)  and the head of vector  \overrightarrow j   is on the y-axis at  (0, 1).  In the notation for Cartesian vectors, \overrightarrow i = [1, 0] and  \overrightarrow j = [0, 1] |  |

If we resolve     into its horizontal and vertical components, we get two vectors that add to  .

***Translations:***

**单位向量:**

|  |  |
| --- | --- |
| 单位向量  \overrightarrow i   和  \overrightarrow j    是笛卡尔向量的“地基”。单位向量的模长为1。 \overrightarrow i   与  \overrightarrow j    的尾部在原点。向量 \overrightarrow i   的首部在横轴上的点 (1, 0)  ，向量  \overrightarrow j   的首部在纵轴上的点 (0, 1)。用笛卡尔向量的形式表达，我们有 \overrightarrow i = [1, 0] ，\overrightarrow j = [0, 1]。 |  |

如果我们将向量   分解成水平方向和竖直方向的两个分量，我们会得到合向量为该向量的两个互相垂直的向量。

**Dot product / 点积（或点乘）**

**Formal Definition:**

|  |
| --- |
| The dot product for any two vectors \overrightarrow {AC}   and  \overrightarrow {AB}   is defined as the product of their magnitudes multiplied by the **cosine** of the angle between the two vectors when the two vectors are placed in a tail-to-tail position |

**点积的正式定义:**

|  |
| --- |
| 任意两个向量 \overrightarrow {AC}   和  \overrightarrow {AB}   的点积被定义为其模长之积再乘以当两个向量之尾重叠放置时其夹角的**余弦值**。 |

There are some important observations that can be made about this calculation. First, the result of the dot product is always a scalar. Each of the quantities on the formula of dot product is a scalar quantity, and so their product must be a scalar. Second, the dot product can be positive, zero, or negative, depending upon the size of the angle between the two vectors.

关于点积的运算，我们能观察出一些重要的规律。首先，点积的运算结果永远是一个标量。点积公式中所代入的量都是标量，所以它们的积也一定是一个标量。 其次，点积的正负值取决于两向量的夹角。

**Projection of One Vector on Another:**

|  |
| --- |
| The vector we obtain by projecting  \overrightarrow v   perpendicularly onto the line  through  \overrightarrow u   is called theVector Projection of  \overrightarrow v   on  \overrightarrow u . |

***Translations:***

**一个向量在另一个向量上的投影:**

|  |
| --- |
| 我们将\overrightarrow v 垂直投影到\overrightarrow u 上从而取得的向量叫做\overrightarrow v 到\overrightarrow u 上的向量投影。 |

**3D Coordinate System / 三维坐标系统**

**n-Dimensional Euclidean Space / n-维欧式空间**

|  |  |  |
| --- | --- | --- |
| **Introduction to 3D Cartesian Vectors:**

|  |
| --- |
| Let vector v represent a vector in space. If vector v is translated so that its tail is at the origin O (0,0,0), then its head is at some point P\left( {{x_1},{y_1},{z_1}} \right).Then the vector v is the called the position vector of the point P which is represented as: v = OP = [x1, y1, z1] |

***Translations:*****3维空间的笛卡尔向量:**

|  |
| --- |
| 令 v 记作为一个空间向量。如果将向量 v 平移，使其尾部与原点O (0,0,0)重合，则其首部位于点 P\left( {{x_1},{y_1},{z_1}} \right)。向量 v 叫做点 P 的位置向量，记作：v = OP = [x1, y1, z1]。 |

 |

|  |
| --- |
| **Cross Product / 叉积（或叉乘）**There are two ways to take the product of a pair of vectors. The first one is **Dot Product** which we discussed in previous lesson. The second method is called the **Cross Product** which we will explore in this learning activity.  The cross product accumulates interactions between different dimensions.向量之间的乘法有两种，第一种是前面所学到过的点积，第二种是我们现在要学的叉积。叉积涉及到不同维度之间的联系。 |
| The name, right-handed system, comes from the fact that we could use our right hand to indicate the direction of the three vectors. If the fingers of your hand curl in the direction of a rotation (through an angle less than 180 degrees) from  \left| {\overrightarrow u } \right|  to \left| {\overrightarrow v } \right|, then your thumb points in the direction of  \overrightarrow u\times\overrightarrow v .右手定则是通过叉积的两个向量的方向来确定叉积所得的第三向量的方向。 将右手手指沿着从 \left| {\overrightarrow u } \right| 到 \left| {\overrightarrow v } \right|（夹角小于180度的一侧）的方向弯曲，此时拇指的指向即为叉积 \overrightarrow u\times\overrightarrow v  的方向。 |

**Equations of Lines in the Plane / 平面内直线的方程**

**Direction Vector:**

|  |
| --- |
| Any non-vertical lines in 2D can be defined using its slope and y-intercept. The slope defines thedirection of a line and the y-intercept defines its exact position, distinguishing it from other lineswith the same slope. A line can be represented in standard form as Ax+By+C=0, whichis called the Scalar Equation, or in Slope-Intercept Form as y=mx+b.In other words, in order to determine a straight line it is enough to specify either of the following sets of information:1. Two points on a line, or2. One point on the line and its directionFor a line l a fixed vector  t\overrightarrow d   is called a direction vector for the line if it is parallel to l.Note that every line has an infinite number of direction vectors that can be represented as t\overrightarrow d  where  \overrightarrow d   is one direction vector for the line and t is a non-zero real number. |

***Translations:***

**方向向量:**

|  |
| --- |
| 任何一个在2维平面的非竖直的直线可以通过其斜率和纵轴截距来表示。斜率代表其方向，而纵轴截距给定了其具体的位置，将其与其他有着相同斜率的直线区分开来。一条直线可以用一种标准的形式来表示： Ax+By+C=0这种形式被叫做标量方程，可以转换成斜率-截距的形式： y=mx+b。换句话说，为了确定一条直线的直线方程或者具体位置，以下两点的检验即为充分条件：1. 在同一条直线上的两点的坐标，或者2. 直线上一点的坐标以及该直线的方向（斜率）。对于一条直线 l ，一个固定的向量 t\overrightarrow d  叫做该直线的方向向量（如果该向量与直线 l 平行）。注意：每条直线有着无穷多个可以表示为t\overrightarrow d 的形式的方向向量，其中 \overrightarrow d 是该直线的任意一个方向向量，而 t 是一个非零实数。 |

**Vector Equation:**

|  |
| --- |
| Using a direction vector for a line to specify the "slant" of the line and one point on the line tofix its location on the coordinate plane. We can develop the vector equation of the line. |
| In general, the vector equation can be written as: |


***Translations:***

**向量方程:**

|  |
| --- |
| 利用方向向量我们可以判断直线的倾斜程度，通过直线上的一点。我们可以确定直线在平面上的位置。我们可以由此推导出直线的向量方程。 |
| 向量方程可以表示为： |



**Parametric Equation:**

Each value of the scalar  in the vector equation corresponds to a point on the line. This scalar is called the parameter for the equation of the line. Another alternative form of an equation of a line that emphasizes how each coordinate is related to the parameter can be easily derived from the position vector form of a vector equation of the line. Therefore, Parametric Equations of the Line through    with direction vector   as    are given as:



The numbers d1 and d2 are called direction numbers of the line.

***Translations:***

**参数方程:**

向量方程中的标量  所能取到的每一个值都对应着直线上的一点。这个标量叫做直线方程的参数。通过考虑参数对横纵坐标值的影响，我们可以从直线的向量方程的位置向量的形式推导出横纵坐标值的参数方程。因此，穿过点并且有着  = [d1, d2]作为方向向量的直线的参数方程即为：

 

实数d1 和 d2 被称作直线的方向数。

**Symmetric Equation:**

Another form of equation of a line evolves from solving Parametric equations for the parameter.

***Translations:***

**对称式方程:**

由通过求解参数方程中的参数而推导出来的另一种直线方程的形式。

**Direction Angles:**

One alternative technique for describing the direction of a line focuses on the direction angles of the line. The direction angles of a line**l**in the plane are the angles alpha **α** and beta **β**, such that:

**0 ≤ α , β ≤ Π**,  between a direction vector of **l** in the upper half-plane (where **y ≥ 0**) and the positive x and y axes.

The direction angles of a line **l** in space are the angles alpha **α**, beta **β**, and gamma **γ**, such that **0 ≤ α , β , γ ≤ Π**, between a direction vector of **l** in the upper half-space (where **z ≥ 0**) and the positive x, y and z axes.

***Translations:***

**方向角:**

一个描述直线方向的方式是直线的方向角。一条在平面内的直线**l**的方向角是满足以下条件的角**α** 和**β**：

**0 ≤ α , β ≤ Π** ，且在位于上半个平面（y **≥ 0**）的**l** 的方向向量和横纵轴的正轴之间。

空间直线 **l** 的方向角是满足以下条件的角 **α**、 **β** 和 **γ** ：

**0 ≤ α , β , γ ≤ Π**, 且在位于上半个空间（**z ≥ 0**）的**l** 的方向向量和x、 y 以及 z 轴之间。

**Normal to a line:**

|  |
| --- |
| A normal (vector) to a line l is a vector  \overrightarrow n   which is perpendicular to the line. This property of normal vectors is what allows us to derive the Cartesian equation of a line. |

***Translations:***

**直线的法向量:**

|  |
| --- |
| 直线 l 的法向量是一个垂直于该直线的向量，记作  \overrightarrow n  。法向量与其对应的直线的垂直性质可以帮助我们推导出直线方程。 |

Equations of **Planes** / **平面**方程

Intersection / **相交**（或者 **交叉** 或者 **交集**）

Intersect / 与···相交

|  |
| --- |
| System of Linear Equations / 线性方程组A system of linear equations is a set of one or more linear equations. When we solve a system of linear equations, we try to find values of the variables that will satisfy the unknowns in each of the equations.一个线性方程组由一个或多个线性方程组成。当我们求解线性方程组时，我们将最终求出的解带入方程组中的每一个等式，等式都会守恒。 |