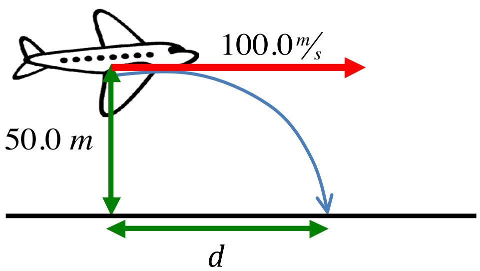
Worksheet 3 – Projectile Motion **SOLUTIONS**

An Alaskan rescue plane drops a package of emergency supplies to a stranded party of explorers. The plane is traveling horizontally at 100.0 m/s at a height of 50.0m above the ground.

- What horizontal distance does the package travel before striking the ground?
- What is the velocity of the package just before it hits the ground (remember this is a vector so direction and magnitude is needed)?

1. *DIAGRAM & LABEL:*



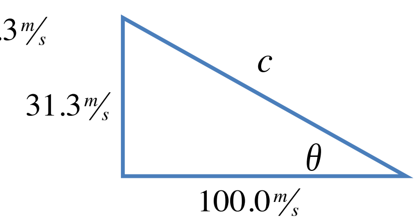
2. *Choose your strategy to solve*
   a. *Use free-fall equation to find the time it took the package to fall to the ground*
   b. *Use the calculated time to find the distance the package travels horizontally*
   c. *Use the calculated time to find the speed the package attains vertically and then use properties of triangles to find the resultant vector for velocity.*

3.

$$t=\sqrt{\frac{2d}{g}} \rightarrow t=\sqrt{\frac{2(50.0\,m)}{9.8\,{}^{m}/_{s^2}}} \rightarrow t=\sqrt{\frac{100.0\,m}{9.8\,{}^{m}/_{s^2}}} \rightarrow t=\sqrt{10.204\,s^2} \rightarrow t=3.19\,s$$

$$d=v_x t \rightarrow d=\left(100\,{}^{m}/_{s}\right)\left(3.19\,s\right) \rightarrow d=319\,m$$

$$v_{yf}=v_{iy}+at \rightarrow v_{yf}=\left(9.8\,{}^{m}/_{s^2}\right)\left(3.19\,s\right) \rightarrow v_{yf}=31.3\,{}^{m}/_{s}$$



$$c^2=a^2+b^2 \rightarrow c=\sqrt{a^2+b^2} \rightarrow c=\sqrt{\left(31.3\,{}^{m}/_{s}\right)^2+\left(100.0\,{}^{m}/_{s}\right)^2}$$

$$c=\sqrt{980+10000} \rightarrow c=\sqrt{10980} \rightarrow c=105\,{}^{m}/_{s}$$

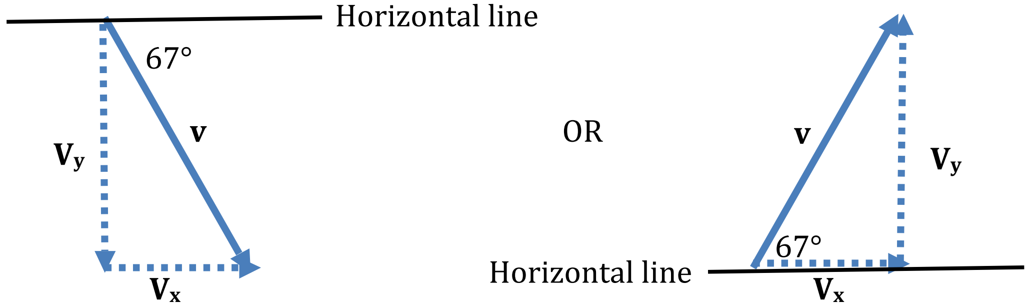
$$\tan\theta=\frac{31.3\,{}^{m}/_{s}}{100.0\,{}^{m}/_{s}} \rightarrow \theta=\tan^{-1}(0.311) \rightarrow 17.3^\circ$$

The resultant vector is 105 m/s at an angle of 17.3° to the horizontal.

A water particle in a stream of water in a fountain takes 0.35 s to travel from spout to receptacle when shot at an angle of 67° and an initial speed of 5.0 m/s.

- What is the vertical distance between the levels of the fountain?

Two scenarios: The fountain is shooting upward into the upper level, or it is shooting downward into the lower level. In both cases, break the velocity into its components to find the vertical velocity.



$$\sin 67° = \frac{opp}{hyp} \rightarrow \sin 67° = \frac{v_y}{5.0\,m/s} \rightarrow (\sin 67°)(5.0\,m/s) = v_y \rightarrow v_y = 4.6\,m/s$$

Use $v_f = v_i + at$ *to find the time it takes for the stream of water to reach the peak. Remember, at the peak, final velocity is zero:*

$$0 = v_y + at \rightarrow -v_i = at \rightarrow \frac{-v_i}{a} = t$$

$$t = \frac{-4.6\,m/s}{-9.8\,m/s^2} \rightarrow t = 0.469s$$

Since it would take the stream of water 0.469s to travel up to its apex and then start to fall down, it can't be shooting up-ward. Remember, it only takes 0.35 s to travel to the next level.

So, it must be shooting down-wards. Use the following equation to find the distance to the next level:

$$d = v_y t + \tfrac{1}{2}at^2 \rightarrow d = (-4.6\,m/s)(0.35s) + \tfrac{1}{2}(-9.8\,m/s^2)(0.35s)^2$$

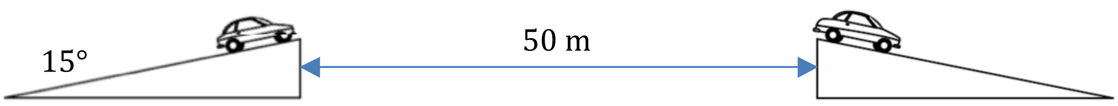
$$d = -1.6m + (-.6m) \rightarrow d = -2.2m$$

Note that the answer is negative because the distance is downwards.

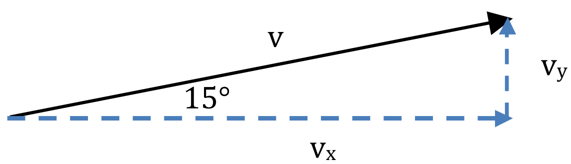
A daredevil attempts to jump a canyon 50 m wide. To do so, she drives her car up a 15° incline. On the other side of the canyon, another 15° incline is built so the car can land properly.

- What minimum speed must she achieve to clear the canyon?

- How long will she be in the air?



This problem is a challenge because it relies on your ability to use properties of triangles and apply a system of equations solution.



$$\sin 15° = \frac{v_y}{v} \quad \rightarrow \quad v \sin 15° = v_y$$

$$\cos 15° = \frac{v_x}{v} \quad \rightarrow \quad v \cos 15° = v_x$$

Horizontal velocity:

$$v_x = \frac{d}{t} \quad \rightarrow \quad v_x t = d \quad \rightarrow \quad t = \frac{d}{v_x}$$

Time in the air to the apex of the flight (remember, at that point the vertical velocity has become zero.

$$v_f = v_i + at \quad \rightarrow \quad 0 = v_y + gt \quad \rightarrow \quad -v_y = gt \quad \rightarrow \quad t = \frac{-v_y}{g}$$

Now, double this time to get the length of time for the entire flight: $t = \frac{-2v_y}{g}$

Combine the two equations together:

$$t = \frac{d}{v_x} \text{ and } \quad t = \frac{-2v_y}{g} \quad \rightarrow \quad \frac{d}{v_x} = \frac{-2v_y}{g} \quad \rightarrow \quad dg = -2v_x v_y \quad \rightarrow \quad v_x v_y = \frac{dg}{-2}$$

This seems like a dead-end, but now combine this equation with the properties of triangles equations above:

$$v_x v_y = \frac{dg}{-2} \qquad \text{and} \qquad v\sin 15° = v_y \quad \text{and} \qquad v\cos 15° = v_x$$

$$(v\sin 15°)(v\cos 15°) = \frac{dg}{-2} \quad \rightarrow \quad v^2 = \frac{dg}{-2(\sin 15°)(\cos 15°)} \quad \rightarrow \quad v = \sqrt{\frac{dg}{-2(\sin 15°)(\cos 15°)}}$$

This seems complicated, but when you multiply sin15° by cos15°, the result is 0.25.

$$v = \sqrt{\frac{dg}{-2(\sin 15°)(\cos 15°)}} \quad \rightarrow \quad v = \sqrt{\frac{dg}{-2(0.25)}} \quad \rightarrow \quad v = \sqrt{\frac{dg}{-0.5}} \quad \rightarrow \quad v = \sqrt{-2dg}$$

Now put in the distance and gravity and calculate the answer

$$v = \sqrt{-2(50.0m)(-9.8 \,{}^{m}\!/\!{}_{s^2})} \quad \rightarrow \quad v = \sqrt{980 \,{}^{m^2}\!/\!{}_{s^2}} \quad \rightarrow \quad v = 31.3 \,{}^{m}\!/\!{}_{s}$$

(Just for reference, 31.3 m/s corresponds to about 70 mph)

You can use the velocity to find the time in the air:

$$v_x = v\cos 15° \quad \& \quad t = \frac{d}{v_x} \quad \rightarrow \quad t = \frac{50m}{(31.3 \,{}^{m}\!/\!{}_{s})(\cos 15°)} \quad \rightarrow \quad t \approx 1.65s$$