The Physics of Roller Coasters Group Activity

Working in groups of 2-3, read the following article on the physics of roller coasters. Afterwards, work together to answer the questions provided below.

The Physics of Roller Coasters

Abstract

The earliest known ride resembling a roller coaster appeared in the seventeenth-century Russia in the form of a large ice slide built on top of a wooden structure. Over the centuries, roller coaster designs became more sophisticated in design and structure. The first roller coaster design that had a loop appeared in the early twentieth century. The roller coaster car had to move fast enough that it could complete the circle without falling. However, the speed required to accomplish this is too fast, and many people were injured on the ride. Today, roller coaster loops are in shape called a clothoid.

Roller Coaster Designs

The earliest rides classified as roller coasters date from the early seventeenth century, in St. Petersburg, Russia. Builders constructed 21m-tall wooden structures and covered them with sheets of ice. Riders climbed stairs at the back of he structure, sat on a sled, and coasted down slopes hundreds of metres long. Later, grooved tracks were added and the sleds were fitted with wheels.

Amusement park designers constructed the first looping roller coaster in the early twentieth century. It had one circular loop. This shape is called a clothed loop (**Figure 1**).

Figure 1 Modern roller coasters have clothoid loops.

Comparing the Two Designs

Using our understanding of circular motion, we can compare the old, circular roller coaster design to the clothoid design and see why the old design is dangerous. To accomplish this, first assume we have two riders, one on each roller coaster design. Next, assume that the height son both designs are the same but the radius of the circular design is twice the radius of the clothoid design at the top (**Figure 2**).

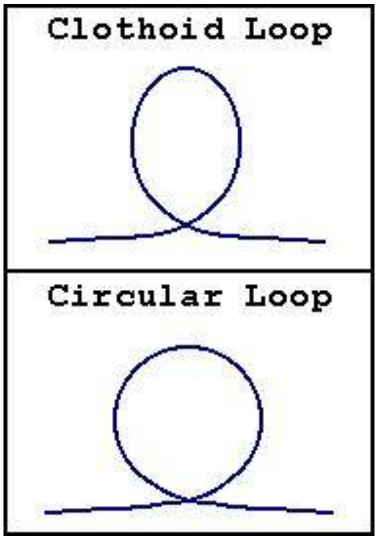

Figure 2 The clothoid design and the circular design.

Assuming the roller coaster car is not attached to the track in any way, the car would need a minimum speed at the top of either type of loop or it would simply fall off at some point. We can calculate the minimum speed for each type of loop and compare them. For simplicity, assume the radius of the circular loop is 15m and the radius of the clothoid at the top is 7.5m.

For the circular loop:

$$\sum F = ma_c$$
$$F_N + mg = \frac{mv^2}{r}$$
$$\text{Set } F_N = 0 \text{ to calculate the minimum speed.}$$
$$0 + \cancel{m}g + = \frac{\cancel{m}v^2}{r}$$
$$g = \frac{v^2}{r}$$
$$v^2 = gr$$
$$v = \sqrt{gr} \quad \text{Take the positive root.}$$
$$= \sqrt{(9.8\, m/s^2)(15m)}$$
$$v = 12m/s$$

For the clothoid loop:

$$v = \sqrt{gr}$$
$$= \sqrt{(9.8\, m/s^2)(7.5m)}$$
$$v = 8.6m/s$$

The minimum speed of the old design roller coasters had to be much faster than the clothoid design roller coaster to clear the loop, even though the heights of both loops are equal (Figure 2). Changing the radius of the loop made the roller coaster safer.

If you were moving at 8.6m/s at the top of a clothoid loop of this design, you would feel weightless for an instant. This results because the normal force drops down to zero at the top and your apparent weight drops to zero. Keep in mind that gravity still acts on you at this point to keep you moving in a circle, but you lose sense of it because you are in free fall.

Roller Coasters and Apparent Weight

Typically, the ride moves much faster than the minimum speed required, and the riders experience normal forces much larger the those in everyday life. This is part of the thrill, of course, but if your apparent weight (normal force) becomes too large or suddenly increases, it can be dangerous. In fact, this is another reason for using the clothoid design.

Now consider an old roller coaster that has a horizontal section of track leading into a circular loop. Once the ride enters the loop, the riders will suddenly experience a centripetal force directed up toward the centre of the loop. The normal force must suddenly increase, not only to overcome gravity but to produce this large centripetal force. This sudden increase in apparent weight can be dangerous to riders.

How does a clothoid design solve this problem? One of the features of a clothoid loop is that the radius of curvature of a clothoid gradually decreases from top to bottom. Even though the radius of curvature of the clothoid at the top is much smaller than that of a circular loop, the same clothoid has a much larger radius of curvature at the bottom. This larger radius of curvature decreases the centripetal force experienced by riders at the bottom of the loop. Since the radius (in the denominator) is larger, the centripetal force required is smaller. This means the normal force required at the start of the loop is reduced and riders do not experience a large sudden increase in apparent weight. In addition, since the radius gradually decreases, the apparent weight is gradually increased, allowing the rider time to adjust without taking away from the excitement of the ride.

Questions

1. A clothoid loop or roller coaster has the same height as a circular loop but half the radius at the top. A rider at the top of either loop typically experiences a net force of 1.5mg. What is the ratio of the speed at the top of the circular loop to the speed at the top of the clothoid?

2. Explain in your own words why the normal force must be set to zero to calculate the minimum possible speed a rider can have at the top of a loop.

3. A 62kg rider is moving at a speed of 22m/s at the bottom of a loop that has a radius of 35m at that point. Determine the normal force acting on the rider due to the seat.