Lesson Big Idea:

Many real-world processes can be modelled with sinusoidal functions. However, the basic sine function usually requires one or more transformations to fit the parameters of the process. One example is the position of the sun above the horizon north of the Arctic Circle in summer. Because the sun does not set during this time, there is no negative value for its position relative to the horizon. As a result, the basic sine function must be adjusted so that the range has no negative values. In this lesson, we will learn how to transform the sine and cosine functions so that we can use them as models for real-world applications.

Ministry Expectations:

2.5, 2.6, 2.7, 2.8

Learning Outcome:

The learning goals of this lesson are to:

- Determine the roles of the parameters a, k, d, and c in functions of the form **y = a sin(k(x-d)) + c** and describe these roles in terms of transformations on the graph of y=sinx.
- Determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given as **y = a sin(k(x-d)) + c** and **y = a cos(k(x-d)) + c**.
- Sketch graphs of **y = a sin(k(x-d)) + c** and **y = a cos(k(x-d)) + c**.

Success Criteria:

By the end of this lesson, you will be able to determine how changing the values of a, c, d, and k affect the graphs of:

f(x) = a sin(k(x - d)) + c and **f(x) = a cos(k(x - d)) + c**.

Teaching Materials:

- Class projector
- Make sure all the learning activities are working on your computer
- Print out exit cards, group activities, and practice worksheets (if necessary)

Assessment and Evaluation:

Assessment FOR Learning

- 4-6E: Transformations Exploration
- 4-6F: Sine Skills Games
- 4-6G: Quiz - Sketching Sine Function
- 4-6H: Check Your Understanding - Practice Worksheet
- 4-6: Group Activity – Investigation: Key Properties of Sine Function – Go Fish Game

Assessment AS Learning

- Exit Cards

Assessment OF Learning

- *4-6I: Evaluation - Transformations (24 Marks)

Instructional Details:

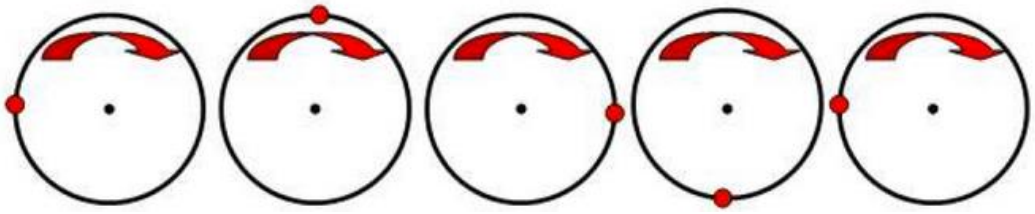
1. Opening:

4-6A: Introduction

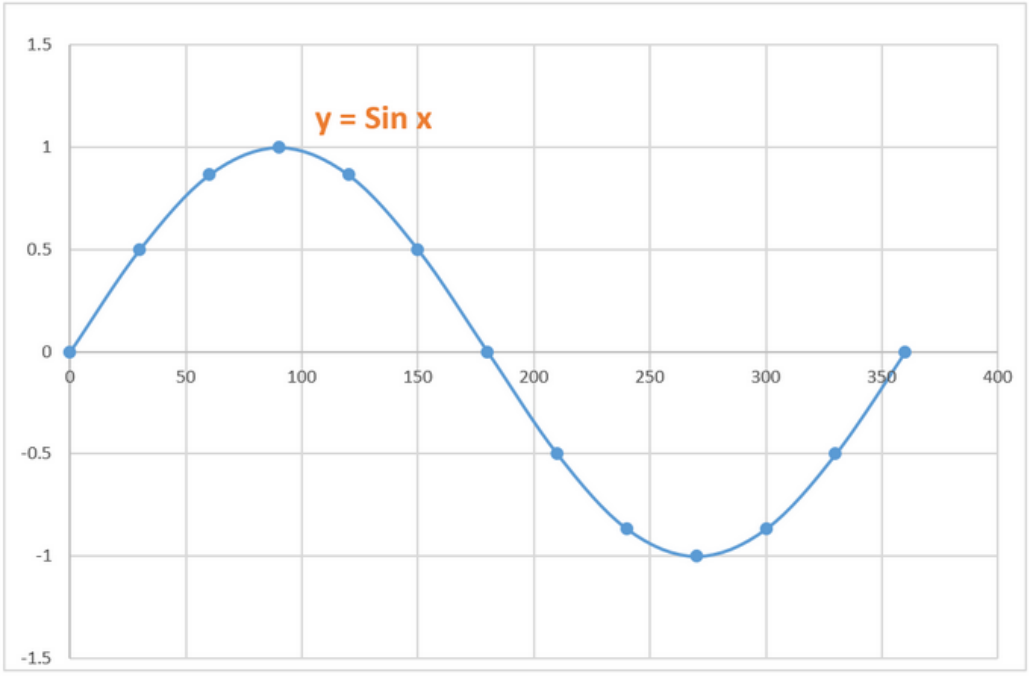
Start the lesson with this introduction activity using real world application example.

- Electrical power is delivered to our homes continuously using a system that can be modelled by a sine function. Each second, the direction of the electron flow (and the voltage) cycles 60 times. If you were to measure the voltage very carefully you would find that it varies, and if graphed it would produce a sine curve. In this learning activity, we will get a chance to see how we can use the sine function to model activities that occur all around us.
- We will begin with a situation that is easy to visualize and easy to model: a wheel. Imagine a wheel that is 2 metres in diameter. To measure the behaviour we will

measure the height of a mark on the wheel. The wheel does not roll but rotates. Examine the red mark on the wheel. As the wheel rotates, the mark changes height. In the first figure, it starts out at the same height as the axle, which we will call 0m.



- Compare this with the second figure. The height is now 1m higher that the axle (remember it is a 2m diameter wheel). In figure 3 the height of the mark is zero again. In figure 4, the height has changed to 1m below the axle. Finally, in figure 5, the height of the mark has returned to 0m again.
- If we were to create a graph of height of the mark vs the angle of rotation, we would have a graph of sin(x) like the one pictured below:



- Review the key properties of sine function: period, amplitude, axis of the curve, max, min, intercepts, domain and range.

- Conclude this activity using the follow up questions below:

Q1. How would the properties of the function change if we were analyzing a wheel with diameter of 4m?

Ans: The amplitude would become 2; the minimum becomes -2 and the maximum +2.

Q2. How would the function itself change if we were analyzing a wheel with diameter of 4m?

Ans: All the y-values would be two times greater, so we would write the function as $y = 2\sin(x)$.

Q3. Suppose we raised the wheel up by a height of 2m. We are not going to change the spot we measure the height from however. How would the properties of the function change if we were analyzing the original wheel with diameter of 2m?

Ans: The maximum would be 3 and the minimum would be 1. The range changes to $1 \leq y \leq 3$.

Q4. How would the function change itself?

Ans: All the y-values would have two added to them, so we would write the function as $y = \sin(x) + 2$.

All these questions are related to some kind of transformation to the graph of a sine function. The transformations that apply to algebraic functions also apply to trigonometric functions e.g. sine and cosine. In the next learning activities, we will explore and investigate transformations of sine and cosine functions in depth.

2. Minds On:

4-6: Group Activity – Key Properties of Sine Functions: Go Fish Game

The purpose of this activity is for students to develop an understanding of key properties of a sine function using a Go Fish. In the game of Go Fish, each player is dealt a number of cards. In turns, each player asks others for cards with the same key properties as one he/she is holding in his/her own hand. When a player collects all four cards representing a key property, the set is put down in front of them. The player with the most sets in the end wins.

The set contains **40 cards**, four cards for each key property. Each property has a card depicting:

- Property Name
- Visual Image (e.g. graph)
- Value or Brief Description
- Definition

Instructions:

- **Print out** the activity and cut each page into 10 cards (equal sizes).
- The ten properties are depicted in the four pages (each property is in the same relative position on each page).
- Split the class in groups. No need to split, if you have less than 10 students.
- Explain the rules of Go Fish game and hand over 40 cards to each group (each player is dealt a number of cards. In turns, each player asks others for cards with the same key properties as one he/she is holding in his/her own hand. When a player collects all four cards representing a key property, the set is put down in front of them. The player with the most sets in the end wins).

Teacher should circulate among pairs and individuals during the activity to ensure that conversations are on-topic, students are encouraging one another, and everyone in the group is contributing.

3. Presentation / Action:

4-6B: Stretches and Translations

- In the following learning activity, we will learn about stretching and translations. We will determine how changing the values of the parameters affect the graph of y = sinx. Please be aware, that, depending on your Internet connection speed, all clips on this page may take a few minutes to download.
- Open the activity on the projector.
- Watch the animation as a group. This animation consists of short videos.
- Review the parameters and write them on the board.
- Watch the video demonstration of translations and stretching using examples as a group.
- Ask students if they have any questions.
- Stop at Its Your Turn slide. This is the part of the activity where students must participate. Click on START to continue.
- Pick a student to answer each question. There are 7 questions in total.
- Make sure to read the feedback together.
- Conclude this activity by summarizing the parameters of the sine and cosine functions.

Reference Point

- Refer to “4-6: Differentiation Resources” on Moodle for video demonstration and links for a variety of examples.

4-6C: Putting it All Together

- Introduce the activity:

  The highest tides in the world are found in the Bay of Fundy. Tides in one area of the bay cause the water level to rise to 6m above average sea level and to drop to 6m below average sea level. The tide completes one cycle approximately every 12 hours. The depth of the water can be modelled by a sine function. In this learning activity, we will perform combinations of transformations in the following order:

  1. Expansions and compressions (vertical and horizontal)
  2. Reflections (along x-axis only)
  3. Translations (vertical and horizontal)

  Functions of the form **f(x) = a sin(k(x - d)) + c** and **f(x) = a cos(k(x - d)) + c** can be graphed by applying the appropriate transformations to the graphs of y = sinx and y = cosx, respectively, one at a time, following the order of operations (multiplication and division before addition and subtraction) for all vertical transformations and for all horizontal transformations. The horizontal and vertical transformations can be completed in either order.

- Open the activity on the projector and review the parameters.
- Do example 1 on the board.
- Stop at example 2 and ask student volunteers to solve example 2 and 3 on the board with your help. Share the feedback once completion.
- Discuss the five-points method using the coordinate and graphs.
- Watch the video solutions of example 4 and 5 as a group on the projector.
- Give opportunity to students to solve example 6 and 7 on the board with your assistance. If students are struggling then do the questions on the board.
- Ask if they have any questions.

4-4C: Summary - Key Properties

These animations will summarize the basic properties of transformations and will make the connection between the parameters and the transformations. These animations have audio component. Watch them as a group on projector.

4. Consolidate / Debrief:

1. Home Activity or Further Classroom Consolidation

4-6E: Check Your Understanding - Transformations Exploration

This animation will give students an opportunity to comprehend the understanding of transformations by solving 15 questions. Students can do this individually on their own during the class time or it can be assigned as homework.

4-6F: Check Your Understanding - Sine Skills Games

In the following animation, students will sketch the sine function. Please click on the START to continue. Students can do this individually on their own during the class time or it can be assigned as homework.

4-6G: Quiz - Sketching Sine Function

This animation is a short quiz related to the sketching of the sine function. Please click on the START to continue. Students can do this individually on their own during the class time or it can be assigned as homework.

4-6H: Check Your Understanding - Practice Worksheet

In this activity, students will complete a practice worksheet. Students can do this individually on their own during the class time or it can be assigned as homework.

Reference Point

Refer to "4-6: Differentiation Resources" on Moodle for extra practice worksheets.

2. Exit Card

Write the following question on the board. Students can solve this question at the end of the class or they can take it home. Make sure to discuss the solutions with the class.

Write down the five points after complete transformation:

$$y = \frac{3}{4}\sin\left(5x - \frac{p}{4}\right) + 6$$

3. Evaluation

***4-6I: Evaluation - Transformations (24 Marks)**

- Download the assignment and print them out for the class.
- Let the students know that this is a formal evaluation that will be used to grade their performance regarding specific learning expectations. Please ensure that they have reviewed the learning activities in this lesson before attempting this evaluation.
- Write it on the board that the assignment is worth 24 Marks and share the distribution of marks based on the KICA category. (Knowledge – 24 Marks)
- Remind the students that the assignment will be marked by the course teacher using the marking scheme. Go through the marking scheme with the class.
- Also, remind them about the basic procedure of how to name the file.
- Ask if they have any questions related to Moodle submission or PDF conversion.