



Lesson Big Idea:

One of the most useful characteristics of trigonometric functions is their periodicity. For example, the times of sunsets, sunrises, and the comet appearances; seasonal temperature changes; the moment of waves in the ocean; and even the quality of a musical sound can be described using trigonometric functions. Mathematicians and scientists use the periodic nature of trigonometric functions to develop mathematical models to predict many natural phenomena. In this lesson, students will solve trigonometric equations algebraically and graphically.

Ministry Expectations:

2.7, 3.4

Learning Outcome:

The learning goal of this lesson are to Investigate how to solve both linear and quadratic trigonometric equations which will require the use of factoring, related acute angles, the CAST rule, periods of trigonometric functions, and trigonometric identities.

Success Criteria:

By the end of this lesson, you will be able to solve linear and quadratic trigonometric equations.





Assessment and Evaluation:

Assessment FOR Learning

- 2-6B: Check Your Understanding Worksheet A (External link)
- 2-6C: Check Your Understanding Worksheet B (External link)
- 2-6D: Check your Understanding Practice Worksheet
- 2-6: Group Activity

Assessment AS Learning

- Graphic Organizer Exit Card
- 2-6: Group Activity

Assessment OF Learning

• *2-6BE Assignment - Trigonometric Equations (36 Marks)

Teaching Materials:

- Class projector
- Make sure all the learning activities are working on your computer
- Print out exit cards

Instructional Details:

- 1. Opening of the Lesson:
- Start the lesson with the review in from the previous lesson.
- Ask students if they have any questions.





2. Minds On:

Group Activity: Trigonometric Equations - Matching Cards

- The focus of this activity is to make connections between a trigonometric equation and a polynomial equation.
- Prepare this activity before the beginning of the class.
- Follow the instructions and guidelines provided below. That guide is also available to download on Moodle activity page. The password is 'mathematics'.
- Solutions are provided at the end of the document.

Instructions:

- Divide the students in groups of 2-3.
- Cut them into 12 pieces (on page 4) and place them in an envelope. In total, you should have 12 cards for each group.
- Distribute one envelope to each group.
- Using those cards, students will determine similar statements. (that is this will find a match between a trigonometric equation and a polynomial equation).
- Once the group determines all the matches. They will need to justify why the equations are similar using the strategy of solving polynomial equations.
- Students will also establish strategy how you will solve the trigonometric equation.
- Remind students NOT to solve the trigonometric equations that is writing in degrees or radians. Students should leave the equation in linear trigonometric form for example:

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Sinx = 1/2
Cosx = -1
Tanx = 1
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Circulate to observe pairs are successful in representing the polynomial equations. Make a mental note to consolidate misconceptions.

You can discuss the reasons for the matches with the whole class (page 5 to 7).





$x^2 = \frac{1}{2}$	$\sin^2 x = \frac{1}{2}$
$x^2 - x = 0$	$\tan^2 x - \tan x = 0$
$x^2 + 3x + 2 = 0$	$\cos^2 x + 3\cos x + 2 = 0$
y + 2xy = 0	$\sin x + 2\cos x \sin x = 0$
xy = x	$\cot x \cos x = \cot x$
xy = 0	$\cos x \sin x = 0$

LESSON & PLANS



Solutions or Reasoning:

$$x^{2} = \frac{1}{2}$$
$$x = \pm \sqrt{\frac{1}{2}}$$
$$x = \pm \frac{1}{\sqrt{2}}$$

$$\sin^2 x = \frac{1}{2}$$
$$\sin x = \pm \sqrt{\frac{1}{2}}$$
$$\sin x = \pm \frac{1}{\sqrt{2}}$$

$$x^{2} - x = 0$$

$$x(x-1) = 0$$

$$x = 0 \quad or \quad x = 1$$

$$\tan^{2} x - \tan x = 0$$

$$\tan x (\tan x - 1) = 0$$

$$\tan x = 0 \quad or \quad \tan x = 1$$





$$x^{2} + 3x + 2 = 0 \qquad \cos^{2} x + 3 = 0 (x + 2)(x + 1) = 0 \qquad (\cos x + 2) = 0 x = -2 \quad or \quad x = -1 \qquad \cos x = -1$$

$$\cos^{2} x + 3\cos x + 2 = 0$$

($\cos x + 2$)($\cos x + 1$) = 0
 $\cos x = -2$ or $\cos x = -1$

$$y + 2xy = 0 \qquad \sin x + 2\cos x \sin x = 0 \\ y(1+2x) = 0 \qquad \sin x(1+2\cos x) = 0 \\ y = 0 \quad or \quad 1+2x = 0 \\ x = -\frac{1}{2} \qquad \cos x = -\frac{1}{2}$$

$$xy = x$$

$$xy = x$$

$$xy - x = 0$$

$$x(y - 1) = 0$$

$$y - 1 = 0$$

$$y = 1$$

$$cot x cos x = cot x$$

$$cot x cos x - cot x = 0$$

$$cot x (cos x - 1) = 0$$

$$cot x = 0 or cos x - 1 = 0$$

$$cos x = 1$$

$$xy = 0 \qquad \cos x \sin x = 0$$

$$x = 0 \quad or \quad y = 0 \qquad \cos x = 0 \quad or \quad \sin x = 0$$





3. Presentation / Action:

Open the learning activity 2-6A on Moodle.

2-6A: Trigonometric Equations

- Begin this activity by having a brief discussion about learning and investigating how to solve trigonometric equations. Trigonometric equations can be solved algebraically by hand or graphically with technology. Because of their periodic nature, trigonometric equations have an infinite number of solutions. We usually want solutions within a specific interval.
- Allow students time to read and take notes of the first part of the 2-6A activity. Make sure they write the three points in "What You Need to Know".
- Circulate around the class to provide opportunities to students to ask questions.
- Print out the chart on page 8 for all students. Students may refer to this chart to get the exact values for the angles.
- Share the following four ways of solving a trigonometric equation. Do all the four types on the board.
- Detailed solutions are provided on page 9 along with check points where you ask students to provide a solution.

Solve by combining like terms:Solve with square roots: $sin x + \sqrt{2} = -sin x$ $3 tan^2 x - 1 = 0$ Solve by factoring:Quadratic type: $csc^4 x - 4 csc^2 x = 0$ $2 sin^2 x - sin x - 1 = 0$



















Check Point! - Ask students to provide solution

1.
$$3 \sec^2 x - 4 = 0$$

 $\Re(x = \frac{4}{3})$
 $\Re(x = \pm 2)$
 $\sqrt{3}$
 $\cos x = \pm \sqrt{3}$
 $\chi = \cos^{-1}(\pm \sqrt{3})$
 $\sqrt{3}$
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(cos, sin)

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(1, 0)

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(0, 1)

3π/2

(0, -1)

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Example 3 Solve by factoring

 $\csc^4 x - 4\csc^2 x = 0$ (0, 1) (¹/₂, (cos, sin) (-1/2, $Csc^2x(csc^2x-4)=0$ $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ CSC=X=0 V Csc2x-4=0 (-1, 0) csc x = 0 $csc^{2}x = 4$ $sin^{2}x = 4$ $sin^{2}x = 4$ $sin^{2}x = 4$ sin x = 1 $sin x = \pm 1$ CSC X=0 3π/2 x= # ±km 6 x= 5 # ±km X=SIN-1/11

Check Point! – Ask students to provide solution

 $(0, 1) (\frac{1}{2},)$ 2. $2\sin^4 x - \sin^2 x = 0$ (cos, sin) $Sln^{2}x(2\sin^{2}x-1)=0$ $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$ (-1, 0) $Sin^2 x = 0$ $2 sin^2 x - 1 = 0$ $(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$ 3π/2 Sin x=0 sin k= 1 $(-\frac{1}{2})$ (0, -1) K=Sin-'(0) Sin K=±L Y= TKI X=sin-1/==





Example 4 Quadratic type



Check Point! - Ask students to provide solution

 $2 \cot^4 x - \cot^2 x - 15 = 0$ 3. $M = Cot^2 x$ $2m^2 - m - 15 = 0$ (2m + 5)(m - 3) = 0(cos, sin) (0, 1) (-1/2 (3) $(a_{10+}+5)(m-3)=0$ $(2 \cot^{2} x + 5)(\cot^{2} x - 3)=0$ $(a_{10+}+5)(\cot^{2} x - 3)=0$ $(a_{10+}+5)(\cot^{2}$ (-1/3, 1 (-1, 0) (-13, -1 3π/2





Go back to the learning activity 2-6A on Moodle.

2-6A: Trigonometric Equations

- This animation consists of a variety of examples from different types.
- Allow students to solve these questions on their own. Ensure to remind them to refer to the chart to get an exact value and seeking for all possible cases.
- It is recommended to write the examples (from the animation) on the board one by one and ask students to solve them. Upon completion, you can share the answers from the animation. Repeat the pattern for the first 5 examples.
- Circulate around the class to ensure students are focused and staying on topic.
- Example 6 is related to the application. Work on that example on the board with the class.
- Do example 7, and 8 on the board. They are related to the double angle.
- Write example 9 on the board and ask students to solve it on their own. Share the detailed solution if students are struggling to comprehend.
- Example 11 and example 12 are unique in a sense since they use identities to simplify. Do example 11 on the board and ask students to work on example 12 on their own.
- Watch the video as a group as a summary.

Reference Point

Refer to "2-6: Resources for Differentiation" on Moodle for a variety of examples or videos.





4. Consolidate / Debrief:

1. Home Activity or Further Classroom Consolidation

2-6B: Check Your Understanding - Worksheet A (External link)

In this activity, students will use the given link to practice questions. Students can do this individually on their own during the class time or it can be assigned as homework. Please remind the students that the solutions are provided on the page.

2-6C: Check Your Understanding - Worksheet B (External link)

In this activity, students will use the given link to practice questions. Students can do this individually on their own during the class time or it can be assigned as homework. Please remind the students that the solutions are provided on the page.

2-6D: Check your Understanding - Practice Worksheet

In this activity, students will complete a practice worksheet. Students can do this individually on their own during the class time or it can be assigned as homework. Please remind the students that the solutions are provided on the page.

Reference Point

Refer to "2-3: Resources for Differentiation" on Moodle for extra practice worksheets for enhanced learners.

2. Evaluation – Home Activity

*2-6BE Assignment - Trigonometric Equations (36 Marks)

- Download the assignment and print them out for the class.
- Let the students know that this is a formal evaluation that will be used to grade their performance regarding specific learning expectations. Please ensure that they have reviewed the learning activities in this lesson before attempting this evaluation.
- Write it on the board that the assignment is worth 36 Marks and share the distribution of marks based on the KICA category. (Inquiry – 36 Marks)
- Remind the students that the assignment will be marked by the course teacher using the marking scheme. Go through the marking scheme with the class.
- \circ $\;$ Also, remind them about the basic procedure of how to name the file.
- Ask if they have any questions related to Moodle submission or PDF conversion.





3. Exit Card – Putting Trigonometric Equation to Work!

- Write the following question on the board. If you are running out of time, then ask students to bring the solution to the following class.
- Exit cards require students to respond to questions or prompts on a piece of paper that they will pass in to you before they leave class. These cards provide you with immediate information that you can use to assess students' understanding or gather feedback on your teaching.

Question:

When batted, a baseball leaves the bat at an angle of θ with the horizontal and an initial velocity of v₀ = 100 feet per second. An outfielder catches the ball 300 feet from home plate. Find θ , given that the range, R, of the projected baseball is modeled by the equation.

$$R = \frac{\left(v_0\right)^2}{32} \sin 2\theta$$